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which is the condition that the equations of geodesics admit the quadratic integral $A_{ij}\frac{dx^i}{ds}\frac{dx^j}{ds} = \text{const.}$ These results are due to Levi-Civita (*Annali*, **24,** 1896, pp. 273, 276; also *Annalen*, **54,** 1901, p. 188), who has found all the Riemann spaces which admit of geodesic representation. When in particular it is assumed that S_n and S_n' possess corresponding orthogonal systems, and these are taken as parametric we have

$$\Gamma^i_{ii} = \frac{1}{2g_{ii}}\frac{\partial g_{ii}}{\partial x^i}, \Gamma^i_{ij} = \frac{1}{2g_{ii}}\frac{\partial g_{ii}}{\partial x_j}, \Gamma^j_{ii} = -\frac{1}{2g_{jj}}\frac{\partial g_{ii}}{\partial x_j}, \Gamma^k_{ij} = 0,$$

and the equations (3.4) and (3.6) are readily solved; giving

$$g_{ii} = \prod_{j=1}^{n}{}' \left|\psi_j - \psi_i\right|, g'_{ii} = \frac{C}{\prod\limits_{j=1}^{n}(\psi_i + c)}\,\frac{g_{ii}}{\psi_i + c},$$

where $j \neq i$ in the first product, ψ_i is an arbitrary function of x^i alone, and c and C are arbitrary constants (Pirro, *Rend. Palermo*, **9,** 1895, p. 169; also, Levi-Civita, l.c., p. 287).

NUMBER OF SUBSTITUTIONS OMITTING AT LEAST ONE LETTER IN A TRANSITIVE GROUP

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In an article published in *Amer. J. Math.*, **26,** 1904, p. 1, H. L. Rietz proved the theorem that in "a primitive group G of degree n of composite order g there are more than $g/(x + 1)$ substitutions of degree less than n, where x is the number of systems of intransitivity of the sub-group which leaves a given letter fixed." If G is a transitive group which is not necessarily primitive this subgroup G_1 is of degree $n - \alpha$, where α is not necessarily unity, and may be supposed to have α transitive constituents of degree 1 in addition to its transitive constituents whose degrees exceed 1. If $x + 1$ represents the total number of these transitive constituents the theorem quoted above remains true for every possible transitive group except the regular groups, as may be inferred from an abstract which appeared in *Bull. Amer. Math. Soc.*, **8,** 1902, p. 17, where the exception noted above is, however, not stated.

The object of the present note is to furnish a very elementary proof of this general theorem, as follows: If each of the substitutions of G_1 involves the average number of letters, viz. $n - x - 1$, the number of substitutions of G which omit at least one letter is $ng_1 \div (x + 1) = g \div (x + 1)$ g_1 being the order of G_1. This case evidently presents itself only when G is regular. In all other cases this number of substitutions must be greater since each such case can be constructed by taking successively one letter away from some one substitution in the idealized G_1' in which all the substitutions including the identity are of the same degree, and adding it to another. By each such operation the number of substitutions which omit at least one letter is increased, since the number of such substitutions is thereby first diminished by $\frac{n}{k} - \frac{n}{k+1} = \frac{n}{k(k+1)}$, where $k \geqq x + 1$, and then increased by $\frac{n}{l-1} - \frac{n}{l} = \frac{n}{l(l-1)}$, where $l \geqq x + 1$. As each such operation increases the total number of substitutions which omit at least one letter this number must be increased by the totality of such operations which lead from the idealized G'_1 to the real G_1. It should be noted that the increase due to one such operation may be a fractional substitution.

In the special case when G is multiply transitive G_1 must be transitive and of degree $n - 1$. In this case $g/(x + 1) = g/_2$ and by reducing one substitution of G'_1 to the identity we increase the number of substitutions whose degree is less than n by

$$\frac{n(n-2)}{2} - \frac{n}{2} + 1.$$

If G_1 contains substitutions whose degree is less than $n - 2$ the number of substitutions of degree $n - 1$ must be still larger. Hence it results directly that in a multiply transitive substitution group of degree n the number of substitutions of degrees $n - 1$ and $n - 2$ cannot be less than

$$\frac{g}{2} + \frac{n(n-3)}{2},$$

and this is the exact number of such substitutions when G involves no substitution besides the identity whose degree is less than $n - 2$ and only then.

The main theorem of this note may be stated as follows: *In every non-regular transitive substitution group of degree n the number of the substitutions which omit at least one letter exceeds g/x where x is the number of the transitive constituents in a subgroup composed of all the substitutions which omit one letter regarded as a group of degree n.* One reason for emphasizing this matter here is that the theorem is stated incorrectly even for the special case when the group is primitive on page 357 of Cajori's *History of*

Mathematics, 1919, as a consequence of a misprint in my "second report on recent progress in the theory of groups of finite order," *Bull. Amer. Math. Soc.*, **9**, 1903, p. 118.

PHOTIC ORIENTATION IN INSECTS

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The idea that orientation in organisms is dependent upon the relation in the physical and chemical states produced by the stimulating agent in symmetrically located receptors and the relation in the activity of the locomotor apparatus on opposite sides is widely accepted. This idea extends back to the botanist, Ray (1693), it was applied to lower animals by Verworn in 1894 and to higher animals including insects by Bohn in 1904 and Loeb in 1906. It has been referred to as the difference of intensity theory, the continuous action theory, the tropism theory, the Verworn theory, the Ray-Verworn or DeCandolle theory, the muscle-tension theory and Loeb's muscle-tension theory. We shall call it the Ray-Verworn theory.

Referring to insects, Bohn, Loeb, Garrey and others maintain that light affects the tonus of the muscles in proportion to the intensity of the illumination of the receptors with which they are connected and in such a way that when the eyes on opposite sides are unequally illuminated, the legs on one side move faster than those on the other resulting in turning until the eyes are equally illuminated, when the tonus of the legs on opposite sides becomes equal and the turning ceases. These authors maintain that whenever insects are oriented, the receptors on opposite sides receive the same amount of stimulating energy. They hold that orientation is the result of balanced action in the two halves of the body.

There are numerous reactions involved in the process of orientation of insects and other animals which are not in accord with these views. Some of these follow.

If one eye of the drone-fly, Eristalis tenax, is covered it deflects strongly toward the functional eye making circus movements, but after a certain time it orients fairly accurately, especially in a well defined beam of light. Similar results have been obtained in observations on various other insects by Radl, (1903), Holmes (1905), Carpenter, (1908), Dolley (1916), and Minnich (1919). The eyes on opposite sides are not equally stimu-